Year 8

Term	Lessons
Autumn 1	20
Autumn 2	18
Spring 1	18
Spring 2	15
Summer 1	15
Summer 2	15
Total:	101

Key Areas

- Apply the four operations with negative numbers
- Convert numbers into standard form and vice versa
- Apply the multiplication, division and power laws of indices
- Convert between terminating decimals and fractions
- Find a relevant multiplier when solving problems involving proportion
- Solve problems involving percentage change, including original value problems
- Factorise an expression by taking out common factors
- Change the subject of a formula when two steps are required
- Find and use the nth term for a linear sequence
- Solve linear equations with unknowns on both sides
- Plot and interpret graphs of linear functions
- Apply the formulae for circumference and area of a circle
- Understanding risk and calculate theoretical/experimental probabilities for single events
- Use Pythagoras' Theorem
- Venn diagrams & Set notation
- Ratio & Proportion
- Charts & Sampling

Essential knowledge

- Know how to write a number as a product of its prime factors
- Know how to round to significant figures
- Know the order of operations including powers
- Know how to enter negative numbers into a calculator
- Know that $a^0 = 1$
- Know percentage and decimal equivalents for fractions with a denominator of 3, 5, 8 and 10
- Know the characteristic shape of a graph of a quadratic function
- Know how to measure and write bearings
- Know how to identify alternate angles
- Know how to identify corresponding angles
- Know how to find the angle sum of any polygon
- Know that circumference = $2\pi r = \pi d$
- Know that area of a circle = πr^2
- Know that volume of prism = area of cross-section × length
- Know to use the midpoints of groups to estimate the mean of a set of grouped data
- Know that probability is measured on a 0-1 scale
- Know that the sum of all probabilities for a single event is 1
- Know how to use Pythagoras' Theorem

Autumn 1 - Number Recap & Indices	MATHSLINK 8C CHAPTERS 1	4 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Calculation progression map

- apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative
- use conventional notation for priority of operations, including brackets, powers, roots and reciprocals
- Laws of indices

Return to overview

Possible themes	Possible key learning points
• Calculate with negative numbers • Apply the correct order of operations • Use and apply the laws of indices	• Subtract a number from a smaller number • Add a positive number to a negative number • Subtract a positive number from a negative number • Add & subtract a negative number • Multiply a positive number by a negative number • Multiply a negative number by a negative number • Divide a positive number by a negative number • Divide a negative number by a negative number • Square and cube positive and negative numbers • Use a scientific calculator to calculate with negative numbers • Use a scientific calculator to calculate with fractions, both positive and negative • Understand how to use the order of operations including powers • Understand how to use the order of operations including roots

Prerequisites	Mathematical language	Pedagogical notes

• Fluently recall and apply multiplication facts up to 12 × 12 • Know and use column addition and subtraction • Know the formal written method of long multiplication • Know the formal written method of short division • Apply the four operations with fractions and mixed numbers • Convert between an improper fraction and a mixed number • Know the order of operations for the four operations and brackets	Negative number Directed number Improper fraction Top-heavy fraction Mixed number Operation Inverse Long multiplication Short division Power Indices Roots	Pupils need to know how to enter negative numbers into their calculator and how to interpret the display. The grid method is promoted as a method that aids numerical understanding and later progresses to multiplying algebraic statements. NRICH: Adding and subtracting positive and negative numbers NRICH: History of negative numbers NCETM: Departmental workshop: Operations with Directed Numbers NCETM: Glossary **Common approaches** *Teachers use the language 'negative number', and not 'minus number', to avoid confusion with calculations* *Every classroom has a negative number washing line on the wall* *Long multiplication and short division are to be promoted as the 'most efficient methods'.* *If any acronym is promoted to help remember the order of operations, then BIDMAS is used as the I stands for indices.*
Reasoning opportunities and Extension questions	**Suggested activities**	**Possible misconceptions**
• Convince me that -3 - -7 = 4 • Show me an example of a calculation involving addition of two negative numbers and the solution -10. And another. And another … • Create a Carroll diagram with 'addition', 'subtraction' as the column headings and 'one negative number', 'two negative numbers' as the row headings. Ask pupils to create (if possible) a calculation that can be placed in each of the four positions. If they think it is not possible, explain why. Repeat for multiplication and division.	KM: Summing up KM: Developing negatives KM: Sorting calculations KM: Maths to Infinity: Directed numbers Standards Unit: N9 Evaluating directed number statements NRICH: Working with directed numbers **Learning review** KM: 8M1 BAM Task	• Some pupils may use a rule stated as 'two minuses make a plus' and make many mistakes as a result; e.g. -4 + -6 = 10 • Some pupils may incorrectly apply the principle of commutativity to subtraction; e.g. 4 – 7 = 3 • The order of operations is often not applied correctly when squaring negative numbers. As a result pupils may think that $x^2 = -9$ when $x = -3$. The fact that a calculator applies the correct order means that $-3^2 = -9$ and this can actually reinforce the misconception. In this situation brackets should be used as follows: $(-3)^2 = 9$.

Autumn 1 - Algebra	***MATHSLINK 8C CHAPTERS 5***	***4 lessons***

Key concepts (GCSE subject content statements)

The Big Picture: Algebra progression map

- use and interpret algebraic notation, including: a^2b in place of $a \times a \times b$, coefficients written as fractions rather than as decimals
- understand and use the concepts and vocabulary of factors
- simplify and manipulate algebraic expressions by taking out common factors and simplifying expressions involving sums, products and powers
- substitute numerical values into scientific formulae
- rearrange formulae to change the subject

Return to overview

Possible themes	Possible key learning points
• Understand the concept of a factor • Understand the notation of algebra • Manipulate algebraic expressions • Evaluate algebraic statements	• Use and interpret algebraic notation, including: $a^2 b$ in place of $a \times a \times b$, coefficients written as fractions rather than as decimals • Simplify an expression involving terms with combinations of variables (e.g. $3a^2b + 4ab^2 + 2a^2 – a^2b$) • Factorise an algebraic expression by taking out common factors • Simplify expressions using the law of indices for multiplication • Simplify expressions using the law of indices for division • Simplify expressions using the law of indices for powers • Know and use the zero index • Substitute positive and negative numbers into formulae • Change the subject of a formula when one step is required • Change the subject of a formula when two steps are required

Prerequisites	Mathematical language	Pedagogical notes
• Know basic algebraic notation (the rules of algebra) • Simplify an expression by collecting like terms • Know how to multiply a single term over a bracket • Substitute positive numbers into expressions and formulae • Calculate with negative numbers	Product Variable Term Coefficient Common factor Factorise Power Indices Formula, Formulae Subject Change the subject **Notation** See Key concepts above	During this unit pupils should experience factorising a quadratic expression such as $6x^2 + 2x$. Collaborate with the science department to establish a list of formulae that will be used, and ensure consistency of approach and experience. NCETM: Algebra NCETM: Departmental workshop: Index Numbers NCETM: Departmental workshops: Deriving and Rearranging Formulae NCETM: Glossary **Common approaches** *Once the laws of indices have been established, all teachers refer to 'like numbers multiplied, add the indices' and 'like numbers divided, subtract the indices. They also generalise to* $a^m \times a^n = a^{m+n}$*, etc.* *When changing the subject of a formula the principle of balancing (doing the same to both sides) must be used rather than a 'change side, change sign' approach.*
Reasoning opportunities and Extension questions	**Suggested activities**	**Possible misconceptions**
• Convince me $a^0 = 1$. • What is wrong with this statement and how can it be corrected: $5^2 \times 5^4 = 5^8$? • Jenny thinks that if y = 2x + 1 then x = (y – 1)/2. Kenny thinks that if y = 2x + 1 then x = y/2 – 1. Who do you agree with? Explain your thinking.	KM: Missing powers KM: Laws of indices. Some useful questions. KM: Maths to Infinity: Indices KM: Scientific substitution (Note that page 2 is hard) NRICH: Temperature **Learning review** KM: 8M3 BAM Task, 8M7 BAM Task, 8M8 BAM Task	• Some pupils may misapply the order of operation when changing the subject of a formula • Many pupils may think that $a^0 = 0$ • Some pupils may not consider 4ab and 3ba as 'like terms' and therefore will not 'collect' them when simplifying expressions

Autumn 1 - Solving equations and inequalities	MATHSLINK 8C CHAPTERS 7 & 13	6 lessons

Key concepts (GCSE subject content statements)

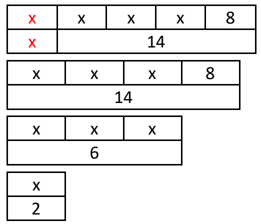
The Big Picture: Algebra progression map

- solve linear equations with the unknown on both sides of the equation
- find approximate solutions to linear equations using a graph

Return to overview

Possible themes	Possible key learning points
• Solve linear equations with the unknown on one side • Solve linear equations with the unknown on both sides • Explore connections between graphs and equations	• Solve linear equations with the unknown on one side when calculating with negative numbers is required • Solve linear equations with the unknown on both sides when the solution is a whole number • Solve linear equations with the unknown on both sides when the solution is a fraction • Solve linear equations with the unknown on both sides when the solution is a negative number • Solve linear equations with the unknown on both sides when the equation involves brackets • Recognise that the point of intersection of two graphs corresponds to the solution of a connected equation

Prerequisites	Mathematical language	Pedagogical notes

• Choose the required inverse operation when solving an equation • Solve linear equations by balancing when the solution is a whole number or a fraction	Algebra, algebraic, algebraically Unknown Equation Operation Solve Solution Brackets Symbol Substitute Graph Point of intersection **Notation** The lower case and upper case of a letter should not be used interchangeably when worked with algebra Juxtaposition is used in place of '×'. 2a is used rather than a2. Division is written as a fraction	This unit builds on the wok solving linear equations with unknowns on one side in Stage 7. It is essential that pupils are secure with solving these equations before moving onto unknowns on both sides. Encourage pupils to 're-present' the problem using the Bar Model. NCETM: The Bar Model NCETM: Algebra NCETM: Glossary  **Common approaches** *All pupils should solve equations by balancing:* $4x + 8 = 14 + x$ $-x \quad -x$ $3x + 8 = 14$ $-8 \quad -8$ $3x = 6$ $\div 3 \quad \div 3$ $x = 2$

Reasoning opportunities and Extension questions	Suggested activities	Possible misconceptions
• Show me an (one-step, two-step) equation with a solution of -8 (negative, fractional solution). And another. And another … • Show me a two-step equation that is 'easy' to solve. And another. And another … • What's the same, what's different: 2x + 7 = 25, 3x + 7 = x + 25, x + 7 = 7 – x, 4x + 14 = 50 ? • Convince me how you could use graphs to find solutions, or estimates, for equations.	KM: Solving equations KM: Stick on the Maths: Constructing and solving equations NRICH: Think of Two Numbers **Learning review** KM: 8M10 BAM Task	• Some pupils may think that you always have to manipulate the equation to have the unknowns on the LHS of the equal sign, for example 2x – 3 = 6x + 6 • Some pupils think if 4x = 2 then x = 2. • When solving equations of the form 2x – 8 = 4 – x, some pupils may subtract 'x' from both sides.

Autumn 1 - Investigating angles | *MATHSLINK 8C CHAPTER 6* | *6 lessons*

Key concepts (GCSE subject content statements) — **The Big Picture**: Position and direction progression map

- understand and use alternate and corresponding angles on parallel lines
- derive and use the sum of angles in a triangle (e.g. to deduce and use the angle sum in any polygon, and to derive properties of regular polygons)

Return to overview

Possible themes	Possible key learning points
• Develop knowledge of angles • Explore geometrical situations involving parallel lines • Plans & Elevations • Tessellations • Interior & Exterior Angles	• Solve missing angle problems involving alternate angles • Solve missing angle problems involving corresponding angles • Use knowledge of alternate and corresponding angles to calculate missing angles in geometrical diagrams • Establish the fact that angles in a triangle must total 180° • Establish the size of an interior angle in a regular polygon • Establish the size of an exterior angle in a regular polygon • Solve missing angle problems in polygons

Prerequisites	Mathematical language	Pedagogical notes
• Use angles at a point, angles at a point on a line and vertically opposite angles to calculate missing angles in geometrical diagrams • Know that the angles in a triangle total 180°	Degrees Right angle, acute angle, obtuse angle, reflex angle Vertically opposite Geometry, geometrical Parallel Alternate angles, corresponding angles Interior angle, exterior angle Regular polygon **Notation** Dash notation to represent equal lengths in shapes and geometric diagrams Arrow notation to show parallel lines	The KM: Perplexing parallels resource is a great way for pupils to discover practically the facts for alternate and corresponding angles. Pupils have established the fact that angles in a triangle total 180° in Stage 7. However, using alternate angles they are now able to prove this fact. Encourage pupils to draw regular and irregular convex polygons to discover the sum of the interior angles = (n – 2) × 180°. NCETM: Glossary **Common approaches** *Teachers insist on correct mathematical language (and not F-angles or Z-angles for example)*
Reasoning opportunities and Extension questions	**Suggested activities**	**Possible misconceptions**
• Show me a pair of alternate (corresponding) angles. And another. And another … • Jenny thinks that hexagons are the only polygon that tessellates. Do you agree? Explain your reasoning. • Convince me that the angles in a triangle total 180°. • Convince me that the interior angle of a pentagon is 540°. • Always/ Sometimes/ Never: The sum of the interior angles of an n-sided polygon can be calculated using sum = (n – 2) × 180°. • Always/ Sometimes/ Never: The sum of the exterior angles of a polygon is 360°.	KM: Alternate and corresponding angles KM: Perplexing parallels KM: Investigating polygons KM: Maths to Infinity: Lines and angles KM: Stick on the Maths: Alternate and corresponding angles KM: Stick on the Maths: Geometrical problems NRICH: Ratty	• Some pupils may think that alternate and/or corresponding angles have a total of 180° rather than being equal. • Some pupils may think that the sum of the interior angles of an n-sided polygon can be calculated using Sum = n × 180°. • Some pupils may think that the sum of the exterior angles increases as the number of sides of the polygon increases.

Autumn 2 – Straight Line graphs — MATHSLINK 8C CHAPTER 7 — 6 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Algebra progression map

- plot graphs of equations that correspond to straight-line graphs in the coordinate plane
- identify and interpret gradients and intercepts of linear functions graphically
- recognise, sketch and interpret graphs of linear functions and simple quadratic functions
- plot and interpret graphs and graphs of non-standard (*piece-wise linear)* functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance and speed

Return to overview

Possible themes	Possible key learning points

• Plot and interpret linear graphs • Plot and quadratic graphs • Model real situations using linear graphs	• Know that graphs of functions of the form $y = mx + c$, $x \pm y = c$ and $ax \pm by = c$ are linear • Plot graphs of functions of the form $y = mx \pm c$ • Plot graphs of functions of the form $ax \pm by = c$ • Find the gradient of a straight line on a unit grid • Find the y-intercept of a straight line • Sketch linear graphs • Distinguish between a linear and quadratic graph • Plot graphs of quadratic functions of the form $y = x^2 \pm c$ • Sketch a simple quadratic graph • Plot and interpret graphs of piece-wise linear functions in real contexts • Plot and interpret distance-time graphs (speed-time graphs) including approximate solutions to kinematic problems

Prerequisites	Mathematical language	Pedagogical notes
• Use coordinates in all four quadrants • Write the equation of a line parallel to the x-axis or the y-axis • Draw a line parallel to the x-axis or the y-axis given its equation • Identify the lines y = x and y = -x • Draw the lines y = x and y = -x • Substitute positive and negative numbers into formulae	Plot Equation (of a graph) Function Formula Linear Coordinate plane Gradient y-intercept Substitute Quadratic Piece-wise linear Model Kinematic, Speed, Distance **Notation** $y = mx + c$	When plotting graphs of functions of the form y = mx + c a table of values can be useful. Note that negative number inputs can cause difficulties. Pupils should be aware that the values they have found for linear functions should correspond to a straight line. NCETM: Glossary **Common approaches** *Pupils are taught to use positive numbers wherever possible to reduce potential difficulties with substitution of negative numbers* *Students plot points with a 'x' and not '•'* *Students draw graphs in pencil* *Use dynamic geometry software to explore graphs of functions*

Reasoning opportunities and Extension questions	Suggested activities	Possible misconceptions
• Draw a distance-time graph of your journey to school. Explain the key features. • Show me a point on this line (e.g. y = 2x + 1). And another, and another … • (Given an appropriate distance-time graph) convince me that Kenny is stationary between 10: 00 a.m. and 10:45 a.m.	KM: Plotting graphs KM: Matching graphs KM: Matching graphs (easy) KM: Autograph 1 KM: Autograph 2 KM: The hare and the tortoise **Learning review** KM: 8M11 BAM Task	• When plotting linear graphs some pupils may draw a line segment that stops at the two most extreme points plotted • Some pupils may think that a sketch is a very rough drawing. It should still identify key features, and look neat, but will not be drawn to scale • Some pupils may think that a positive gradient on a distance-time graph corresponds to a section of the journey that is uphill • Some pupils may think that the graph $y = x^2 + c$ is the graph of $y = x^2$ translated horizontally.

Autumn 2 – Probability 1 — *MATHSLINK 8C CHAPTER 3* — *6 lessons*

Key concepts (GCSE subject content statements)

The Big Picture: Probability progression map

- relate relative expected frequencies to theoretical probability, using appropriate language and the 0 - 1 probability scale
- record describe and analyse the frequency of outcomes of probability experiments using tables
- construct theoretical possibility spaces for single experiments with equally likely outcomes and use these to calculate theoretical probabilities
- apply the property that the probabilities of an exhaustive set of outcomes sum to one

Return to overview

Possible themes	Possible key learning points
• Understand the meaning of probability • Explore experiments and outcomes • Develop understanding of probability	• Know and use the vocabulary of probability • Understand the use of the 0-1 scale to measure probability • List all the outcomes for an experiment, including the use of tables • Work out theoretical probabilities for events with equally likely outcomes • Know that the sum of probabilities for all outcomes is 1 • Apply the fact that the sum of probabilities for all outcomes is 1

Prerequisites	Mathematical language	Pedagogical notes
• Understand the equivalence between fractions, decimals and percentages • Compare fractions, decimals or percentages • Simplify a fraction by cancelling common factors	Probability, Theoretical probability Event Outcome Impossible, Unlikely, Evens chance, Likely, Certain Equally likely Mutually exclusive Exhaustive Possibility space Experiment **Notation** Probabilities are expressed as fractions, decimals or percentage. They should not be expressed as ratios (which represent odds) or as words	This is the first time students will meet probability. It is not immediately apparent how to use words to label the middle of the probability scale. 'Evens chance' is a common way to do so, although this can be misleading as it could be argued that there is an even chance of obtaining any number when rolling a fair die. NRICH: Introducing probability NRICH: Why Do People Find Probability Unintuitive and Difficult? NCETM: Glossary **Common approaches** *Every classroom has a display of a probability scale labeled with words and numbers. Pupils create events and outcomes that are placed on this scale.*

Reasoning opportunities and Extension questions	Suggested activities	Possible misconceptions
• Show me an example of an event and outcome with a probability of 0. And another. And another… • Always / Sometimes / Never: if I pick a card from a pack of playing cards then the probability of picking a club is ¼ • Label this (eight-sided) spinner so that the probability of scoring a 2 is ¼. How many different ways can you label it?	KM: Probability scale and slideshow version KM: Probability loop cards NRICH: Dice and spinners interactive **Learning review** KM: 8M13 BAM Task	• Some pupils will initially think that, for example, the probability of it raining tomorrow is ½ as it either will or it won't. • Some students may write a probability as odds (e.g. 1:6 or '1 to 6'). There is a difference between probability and odds, and therefore probabilities must only be written as fractions, decimals or percentages. • Some pupils may think that, for example, if they flip a fair coin three times and obtain three heads, then it must be more than likely they will obtain a head next.

Autumn 2 – Probability 2	***MATHSLINK 8C CHAPTER 3***	***3 lessons***
Key concepts (GCSE subject content statements)		**The Big Picture**: Probability progression map

- apply systematic listing strategies
- record describe and analyse the frequency of outcomes of probability experiments using frequency trees
- enumerate sets and combinations of sets systematically, using tables, grids and Venn diagrams
- construct theoretical possibility spaces for combined experiments with equally likely outcomes and use these to calculate theoretical probabilities
- apply ideas of randomness, fairness and equally likely events to calculate expected outcomes of multiple future experiments

Return to overview

Possible themes	Possible key learning points
• Explore experiments and outcomes • Develop understanding of probability • Use probability to make predictions	• List all elements in a combination of sets using a Venn diagram • List outcomes of an event systematically • Use a table to list all outcomes of an event • Use frequency trees to record outcomes of probability experiments • Construct theoretical possibility spaces for combined experiments with equally likely outcomes • Calculate probabilities using a possibility space • Use theoretical probability to calculate expected outcomes • Use experimental probability to calculate expected outcomes

Prerequisites	Mathematical language	Pedagogical notes
• Convert between fractions, decimals and percentages • Understand the use of the 0-1 scale to measure probability • Work out theoretical probabilities for events with equally likely outcomes • Know how to represent a probability • Know that the sum of probabilities for all outcomes is 1	Outcome Event Experiment, Combined experiment Frequency tree Enumerate Set Venn diagram Possibility space, sample space Equally likely outcomes Theoretical probability Random Bias, Fairness Relative frequency **Notation** P(A) for the probability of event A Probabilities are expressed as fractions, decimals or percentage. They should not be expressed as ratios (which represent odds) or as words	The Venn diagram was invented by John Venn (1834 – 1923) NCETM: Glossary **Common approaches** *All students are taught to use 'DIME' probability recording charts* *All classes carry out the 'race game' as a simulated horse race with horses numbered 1 to 12*

Reasoning opportunities and Extension questions	Suggested activities	Possible misconceptions
• Show me a way of listing all outcomes when two coins are flipped • Convince me that there are more than 12 outcomes when two six-sided dice are rolled • Convince me that 7 is the most likely total when two dice are rolled	KM: Sample spaces KM: Race game Hwb: Q37, Q79 KM: Stick on the Maths L4HD3 NRICH: Prize Giving (frequency trees)	• Some students may think that there are only three outcomes when two coins are flipped, or that there are only six outcomes when three coins are flipped • Some students may think that there are 12 unique outcomes when two dice are rolled • Some students may think that there are 12 possible totals when two dice are rolled

Autumn 2 – Assessment	***3 lessons***

- One hour non calculator SAT style test
- Self-assessment sheets completed
- Review and self-assessment of performance stuck into books

Spring 1 - Presentation of data	MATHSLINK 8C CHAPTER 3, 11 & 15	4 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Statistics progression map

- interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate graphical representation involving discrete, continuous and grouped data
- use and interpret scatter graphs of bivariate data
- recognise correlation

Return to overview

Possible themes	Possible key learning points
• Explore types of data • Construct and interpret graphs • Select appropriate graphs and charts	• Construct and interpret a grouped frequency table for continuous data • Construct and interpret histograms for grouped data with equal class intervals • Plot a scatter diagram of bivariate data • Interpret a scatter diagram using understanding of correlation

Prerequisites	Mathematical language	Pedagogical notes
• Know the meaning of discrete data • Interpret and construct frequency tables • Construct and interpret pictograms, bar charts, pie charts, tables and vertical line charts	Data Categorical data, Discrete data Continuous data, Grouped data Table, Frequency table Frequency Scale, Graph Axis, axes Scatter graph (scatter diagram, scattergram, scatter plot) Bivariate data (Linear) Correlation Positive correlation, Negative correlation **Notation** Correct use of inequality symbols when labeling groups in a frequency table	The word histogram is often misused and an internet search of the word will usually reveal a majority of non-histograms. The correct definition is 'a diagram made of rectangles whose areas are proportional to the frequency of the group'. If the class widths are equal, as they are in this unit of work, then the vertical axis shows the frequency. It is only later that pupils need to be introduced to unequal class widths and frequency density. Lines of best fit on scatter diagrams are not introduced until Stage 9, although pupils may well have encountered both lines and curves of best fit in science by this time. NCETM: Glossary **Common approaches** *Students collect data about their class's height and armspan when first constructing a scatter diagram*

Reasoning opportunities and Extension questions	Suggested activities	Possible misconceptions
• Show me a scatter graph with positive (negative, no) correlation. And another. And another. • Kenny thinks that 'frequency diagram' is just a 'fancy' name for a bar chart. Do you agree with Kenny? Explain your answer. • What's the same and what's different: scatter diagram, bar chart, pie chart? • Always/Sometimes/Never: A scatter graph shows correlation	KM: Make a 'human' scatter graph by asking pupils to stand at different points on a giant set of axes. KM: Gathering data KM: Spreadsheet statistics KM: Stick on the Maths HD2: Selecting and constructing graphs and charts KM: Stick on the Maths HD3: Working with grouped data	• Some pupils may label the bar of a histogram rather than the boundaries of the bars • Some pupils may think that there are gaps between the bars in a histogram • Some pupils may misuse the inequality symbols when working with a grouped frequency table

Spring 1 - Measuring data	MATHSLINK 8C CHAPTER 3, 11 & 15	6 lessons

Key concepts (GCSE subject content statements) **The Big Picture**: Statistics progression map

- interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate measures of central tendency (median, mean, mode and modal class) and spread (range, including consideration of outliers)
- apply statistics to describe a population

Return to overview

Possible themes	Possible key learning points
• Investigate averages • Explore ways of summarising data • Analyse and compare sets of data	• Find the modal class of set of grouped data • Find the class containing the median of a set of data • Calculate an estimate of the mean from a grouped frequency table • Estimate the range from a grouped frequency table • Analyse and compare sets of data, appreciating the limitations of different statistics (mean, median, mode, range) • Choose appropriate statistics to describe a set of data

Prerequisites	Mathematical language	Pedagogical notes
• Understand the mean, mode and median as measures of typicality (or location) • Find the mean, median, mode and range of a set of data • Find the mean, median, mode and range from a frequency table	Average Spread Consistency Mean Median Mode Range Statistic Statistics Approximate, Round Calculate an estimate Grouped frequency Midpoint **Notation** Correct use of inequality symbols when labeling groups in a frequency table	The word 'average' is often used synonymously with the mean, but it is only one type of average. In fact, there are several different types of mean (the one in this unit properly being named as the 'arithmetic mean'). NCETM: Glossary **Common approaches** *Classroom has a set of statistics posters on the wall* *Students are taught to use mathematical presentation correctly when calculating and rounding solutions, e.g. (21 + 56 + 35 + 12) ÷ 30 = 124 ÷ 30 = 41.3 to 1 d.p.*

Reasoning opportunities and Extension questions	Suggested activities	Possible misconceptions
• Show me an example of an outlier. And another. And another. • Convince me why the mean from a grouped set of data is only an estimate. • What's the same and what's different: mean, modal class, median, range? • Always/Sometimes/Never: A set of grouped data will have one modal class • Convince me how to estimate the range for grouped data.	KM: Swillions KM: Lottery project NRICH: Half a Minute	• Some pupils may incorrectly estimate the mean by dividing the total by the numbers of groups rather than the total frequency. • Some pupils may incorrectly think that there can only be one model class. • Some pupils may incorrectly estimate the range of grouped data by subtracting the upper bound of the first group from the lower bound of the last group.

Spring 1 - Pythagoras' Theorem and Calculating Space — MATHSLINK 8C CHAPTER 14 — 8 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Measurement and mensuration progression map

- identify and apply circle definitions and properties, including: tangent, arc, sector and segment
- calculate arc lengths, angles and areas of sectors of circles
- *calculate surface area of right prisms (including cylinders)*
- calculate exactly with multiples of π
- know the formulae for: Pythagoras' theorem, $a^2 + b^2 = c^2$, and apply it to find lengths in right-angled triangles in two dimensional figures

Return to overview

Possible themes

- Solve problems involving arcs and sectors
- Solve problems involving prisms
- Investigate right-angled triangles
- Solve problems involving Pythagoras' theorem

Possible key learning points

- Know circle definitions and properties, including: tangent, arc, sector and segment
- Calculate the arc length of a sector, including calculating exactly with multiples of π
- Calculate the area of a sector, including calculating exactly with multiples of π
- Calculate the angle of a sector when the arc length and radius are known
- Calculate the surface area of a right prism
- Calculate the surface area of a cylinder, including calculating exactly with multiples of π
- Know and use Pythagoras' theorem
- Calculate the hypotenuse of a right-angled triangle using Pythagoras' theorem in two dimensional figures
- Calculate one of the shorter sides in a right-angled triangle using Pythagoras' theorem in two dimensional figures
- Solve problems using Pythagoras' theorem in two dimensional figures

Prerequisites

- Know and use the number π
- Know and use the formula for area and circumference of a circle
- Know how to use formulae to find the area of rectangles, parallelograms, triangles and trapezia
- Know how to find the area of compound shapes

Mathematical language

Circle, Pi
Radius, diameter, chord, circumference, arc, tangent, sector, segment
(Right) prism, cylinder
Cross-section
Hypotenuse
Pythagoras' theorem

Notation

π
Abbreviations of units in the metric system: km, m, cm, mm, mm², cm², m², km², mm³, cm³, km³

Pedagogical notes

This unit builds on the area and circle work form Stages 7 and 8. Students will need to be reminded of the key formula, in particular the importance of the perpendicular height when calculating areas and the correct use of πr^2. Note: some students may only find the area of the three 'distinct' faces when finding surface area.
Students must experience right-angled triangles in different orientations to appreciate the hypotenuse is always opposite the right angle.
NCETM: Glossary

Common approaches

Students visualize and write down the shapes of all the faces of a prism before calculating the surface area. Every classroom has a set of area posters on the wall.
Pythagoras' theorem is stated as 'the square of the hypotenuse is equal to the sum of the squares of the other two sides' not just $a^2 + b^2 = c^2$.

Reasoning opportunities and Extension questions

- Show me a sector with area 25π. And another. And another …
- Always/ Sometimes/ Never: The value of the volume of a prism is less than the value of the surface area of a prism.
- Always/ Sometimes/ Never: If $a^2 + b^2 = c^2$, a triangle with sides a, b and c is right angled.
- Kenny thinks it is possible to use Pythagoras' theorem to find the height of isosceles triangles that are not right- angled. Do you agree with Kenny? Explain your answer.
- Convince me the hypotenuse can be represented as a horizontal line.

Suggested activities

KM: The language of circles
KM: One old Greek (geometrical derivation of Pythagoras' theorem. This is explored further in the next unit)
KM: Stick on the Maths: Pythagoras' Theorem
KM: Stick on the Maths: Right Prisms
NRICH: Curvy Areas
NRICH: Changing Areas, Changing Volumes

Learning review
KM: 9M10 BAM Task, 9M11 BAM Task

Possible misconceptions

- Some students will work out $(\pi \times r)^2$ when finding the area of a circle
- Some students may use the sloping height when finding cross-sectional areas that are parallelograms, triangles or trapezia
- Some students may confuse the concepts of surface area and volume
- Some students may use Pythagoras' theorem as though the missing side is always the hypotenuse
- Some students may not include the lengths of the radii when calculating the perimeter of an sector

Spring 2 - Calculating space in 2d & 3d shapes	MATHSLINK 8C CHAPTER 2 & 14	6 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Measurement and mensuration progression map

- compare lengths, areas and volumes using ratio notation
- calculate perimeters of 2D shapes, including circles
- identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference
- know the formulae: circumference of a circle = $2\pi r = \pi d$, area of a circle = πr^2
- calculate areas of circles and composite shapes
- know and apply formulae to calculate volume of right prisms (including cylinders)

Return to overview

Possible themes

- Investigate circles
- Discover pi
- Solve problems involving circles
- Explore prisms and cylinders

Possible key learning points

- Know circle definitions and properties, including: centre, radius, chord, diameter, circumference
- Calculate the circumference of a circle when radius or diameter is given
- Calculate the perimeter of composite shapes that include sections of a circle
- Calculate the area of a circle when radius or diameter is given
- Calculate the area of composite shapes that include sections of a circle
- Calculate the volume of a right prism
- Calculate the volume of a cylinder
- Compare lengths, areas and volumes using ratio notation

Prerequisites

- Know how to use formulae to find the area of rectangles, parallelograms, triangles and trapezia
- Know how to find the area of compound shapes

Mathematical language

Circle
Centre
Radius, diameter, chord, circumference
Pi
(Right) prism
Cross-section
Cylinder
Polygon, polygonal
Solid

Notation

π
Abbreviations of units: km, m, cm, mm, mm^2, cm^2, m^2, km^2, mm^3, cm^3, km^3

Pedagogical notes

$C = \pi d$ can be established by investigating the ratio of the circumference to the diameter of circular objects (wheel, clock, tins, glue sticks, etc.) Pupils need to understand this formula in order to derive $A = \pi r^2$.
A prism is a solid with constant polygonal cross-section. A right prism is a prism with a cross-section that is perpendicular to the 'length'.
NCETM: Glossary

Common approaches

The area of a circle is derived by cutting a circle into many identical sectors and approximating a parallelogram
Every classroom has a set of area posters on the wall
The formula for the volume of a prism is 'area of cross-section × length' even if the orientation of the solid suggests that height is required
Pupils use area of a trapezium = $\frac{(a+b)h}{2}$ *and area of a triangle = area =* $\frac{bh}{2}$

Reasoning opportunities and Extension questions

- Convince me $C = 2\pi r = \pi d$.
- What is wrong with this statement? How can you correct it?
  The area of a circle with radius 7 cm is approximately 441 cm^2 because $(3 \times 7)^2 = 441$.
- Convince me that the area of a semi-circle = $\frac{\pi d^2}{8}$
- Name a right prism. And another. And another …
- Convince me that a cylinder is not a prism

Suggested activities

KM: Circle connections, Circle connections v2
KM: Circle circumferences, Circle problems
KM: Circumference searching
KM: Maths to Infinity: Area and Volume
KM: Stick on the Maths: Circumference and area of a circle
KM: Stick on the Maths: Right prisms
NRICH: Blue and White
NRICH: Efficient Cutting
NRICH: Cola Can
Learning review
KM: 8M12 BAM Task

Possible misconceptions

- Some pupils will work out $(\pi \times \text{radius})^2$ when finding the area of a circle
- Some pupils may use the sloping height when finding cross-sectional areas that are parallelograms, triangles or trapezia
- Some pupils may think that the area of a triangle = base × height
- Some pupils may think that you multiply all the numbers to find the volume of a prism
- Some pupils may confuse the concepts of surface area and volume

Spring 2 - Numbers and the number system, Standard Form	MATHSLINK 8C CHAPTER 8	6 lessons

Key concepts (GCSE subject content statements) — **The Big Picture**: Number and Place Value progression map

- use the concepts and vocabulary of prime numbers, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation theorem
- round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures)
- Laws of indices including algebraic expressions
- interpret standard form $A \times 10^n$, where $1 \le A < 10$ and n is an integer

Return to overview

Possible themes

- Identify and use the prime factorisation of a number
- Understand and use the laws of indices
- Simplify algebraic expressions using laws of indices
- Understand and use standard form

Possible key learning points

- Write a number as a product of its prime factors
- Use prime factorisations to find the highest common factor of two numbers
- Use prime factorisations to find the lowest common multiple of two numbers
- Solve problems using highest common factors or lowest common multiples
- Round numbers to a given number of significant figures
- Use standard form to write large numbers
- Use standard form to write small numbers

Prerequisites

- Know the meaning of a prime number
- Recall prime numbers up to 50
- Understand the use of notation for powers
- Know how to round to the nearest whole number, 10, 100, 1000 and to decimal places
- Multiply and divide numbers by powers of 10
- Know how to identify the first significant figure in any number
- Approximate by rounding to the first significant figure in any number

Mathematical language

Prime
Prime factor
Prime factorisation
Product
Venn diagram
Highest common factor
Lowest common multiple
Standard form
Significant figure

Notation
Index notation: e.g. 5^3 is read as '5 to the power of 3' and means '3 lots of 5 multiplied together'
Standard form (see Key concepts) is sometimes called 'standard index form', or more properly, 'scientific notation'

Pedagogical notes

Pupils should explore the ways to enter and interpret numbers in standard form on a scientific calculator. Different calculators may very well have different displays, notations and methods.
Liaise with the science department to establish when students first meet the use of standard form, and in what contexts they will be expected to interpret it.
NRICH: Divisibility testing
NCETM: Glossary

Common approaches
The following definition of a prime number should be used in order to minimise confusion about 1: A prime number is a number with exactly two factors.
The description 'standard form' is always used instead of 'scientific notation' or 'standard index form'

Reasoning opportunities and Extension questions

- Show me two (three-digit) numbers with a highest common factor of 18. And another. And another…
- Show me two numbers with a lowest common multiple of 240. And another. And another…
- Jenny writes $7.1 \times 10^{-5} = 0.0000071$. Kenny writes $7.1 \times 10^{-5} = 0.000071$. Who do you agree with? Give reasons for your answer.

Suggested activities

Use the number 5040 when writing prime factorisations
KM: Ben Nevis
KM: Astronomical numbers
KM: Interesting standard form
KM: Powers of ten
KM: Maths to Infinity: Standard form
Powers of ten film (external site)
The scale of the universe animation (external site)

Learning review
KM: 8M2 BAM Task

Possible misconceptions

- Many pupils believe that 1 is a prime number – a misconception which can arise if the definition is taken as 'a number which is divisible by itself and 1'
- Some pupils may think 35 934 = 36 to two significant figures
- When converting between ordinary and standard form some pupils may incorrectly connect the power to the number of zeros; e.g. $4 \times 10^5 = 400\ 000$ so $4.2 \times 10^5 = 4\ 200\ 000$
- Similarly, when working with small numbers (negative powers of 10) some pupils may think that the power indicates how many zeros should be placed between the decimal point and the first non-zero digit

Spring 2 – Assessment	*3 lessons*
• One hour non calculator SAT style test • Self-assessment sheets completed • Review and self-assessment of performance stuck into books	

Summer 1 – Ratio & Proportional reasoning	***MATHSLINK 8C CHAPTER 12***	***5 lessons***

Key concepts (GCSE subject content statements)

The Big Picture: Ratio and Proportion progression map

- express the division of a quantity into two parts as a ratio; apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)
- identify and work with fractions in ratio problems
- understand and use proportion as equality of ratios
- express a multiplicative relationship between two quantities as a ratio or a fraction
- use compound units (such as speed, rates of pay, unit pricing)
- change freely between compound units (e.g. speed, rates of pay, prices) in numerical contexts
- relate ratios to fractions and to linear functions

Return to overview

Possible themes	Possible key learning points
• Explore the uses of ratio • Investigate the connection between ratio and proportion • Solve problems involving proportional reasoning • Solve problems involving compound units	• Express the division of a quantity into two parts as a ratio • Understand the connections between ratios and fractions • Find a relevant multiplier in a situation involving proportion • Solve ratio problems involving mixing • Solve ratio problems involving comparison • Solve ratio problems involving concentrations • Understand and use compound units • Convert between units of speed • Solve problems involving speed • Solve problems involving rates of pay • Solve problems involving unit pricing

Prerequisites	Mathematical language	Pedagogical notes

• Understand and use ratio notation • Divide an amount in a given ratio	Ratio Proportion Proportional Multiplier Speed Unitary method Units Compound unit **Notation** Kilometres per hour is written as km/h or kmh^{-1} Metres per second is written as m/s or ms^{-1}	The Bar Model is a powerful strategy for pupils to 're-present' a problem involving ratio. NCETM: The Bar Model NCETM: Multiplicative reasoning NCETM: Departmental workshops: Proportional Reasoning NCETM: Glossary **Common approaches** *All pupils are taught to set up a 'proportion table' and use it to find the multiplier in situations involving proportion*
Reasoning opportunities and Extension questions	**Suggested activities**	**Possible misconceptions**
• Show me an example of two quantities that will be in proportion. And another. And another … • (Showing a table of values such as the one below) convince me that this information shows a proportional relationship 6 | 9 10 | 15 14 | 21 • Which is the faster speed: 60 km/h or 10 m/s? Explain why.	KM: Proportion for real KM: Investigating proportionality KM: Maths to Infinity: Fractions, decimals, percentages, ratio, proportion NRICH: In proportion NRICH: Ratio or proportion? NRICH: Roasting old chestnuts 3 Standards Unit: N6 Developing proportional reasoning **Learning review** KM: 8M5 BAM Task	• Many pupils will want to identify an additive relationship between two quantities that are in proportion and apply this to other quantities in order to find missing amounts • Some pupils may think that a multiplier always has to be greater than 1 • When converting between times and units, some pupils may base their working on 100 minutes = 1 hour

Summer 1 – Bearings and scale drawing	MATHSLINK 8C CHAPTER 14	6 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Properties of Shape progression map

- measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of bearings
- identify, describe and construct similar shapes, including on coordinate axes, by considering enlargement
- interpret plans and elevations of 3D shapes
- use scale factors, scale diagrams and maps

Return to overview

Possible themes	Possible key learning points
• Explore enlargement of 2D shapes • Use and interpret scale drawings • Use and interpret bearings • Explore ways of representing 3D shapes	• Use the centre and scale factor to carry out an enlargement with a positive integer scale factor • Find the centre of enlargement • Find the scale factor of an enlargement • Use scale diagrams, including maps • Use the concept of scaling in diagrams • Interpret plans and elevations • Understand and use bearings • Construct scale diagrams involving bearings • Solve geometrical problems using bearings

Prerequisites	Mathematical language	Pedagogical notes
• Use a protractor to measure angles to the nearest degree • Use a ruler to measure lengths to the nearest millimetre • Understand coordinates in all four quadrants • Work out a multiplier given two numbers • Understand the concept of an enlargement (no scale factor)	Similar, Similarity Enlarge, enlargement Scaling Scale factor Centre of enlargement Object Image Scale drawing Bearing Plan, Elevation **Notation** Bearings are always given as three figures; e.g. 025°. Cartesian coordinates: separated by a comma and enclosed by brackets	Describing enlargement as a 'scaling' will help prevent confusion when dealing with fractional scale factors NCETM: Departmental workshops: Enlargement NCETM: Glossary **Common approaches** *All pupils should experience using dynamic software (e.g. Autograph) to visualise the effect of moving the centre of enlargement, and the effect of varying the scale factor.*
Reasoning opportunities and Extension questions	**Suggested activities**	**Possible misconceptions**

• Give an example of a shape and its enlargement (e.g. scale factor 2) with the guidelines drawn on. How many different ways can the scale factor be derived? • Show me an example of a sketch where the bearing of A from B is between 90° and 180°. And another. And another … • The bearing of A from B is 'x'. Find the bearing of B from A in terms of 'x'. Explain why this works. • Provide the plan and elevations of shapes made from some cubes. Challenge pupils to build the shape and place it in the correct orientation.	KM: Outdoor Leisure 13 KM: Airports and hilltops KM: Plans and elevations KM: Transformation template KM: Enlargement I KM: Enlargement II KM: Investigating transformations with Autograph (enlargement and Main Event II). Dynamic example. KM: Solid problems (plans and elevations) KM: Stick on the Maths: plans and elevations WisWeb applet: Building houses NRICH: Who's the fairest of them all? **Learning review** www.diagnosticquestions.com	• Some pupils may think that the centre of enlargement always has to be (0,0), or that the centre of enlargement will be in the centre of the object shape. • If the bearing of A from B is 'x', then some pupils may think that the bearing of B from A is '180 – x'. • The north elevation is the view of a shape from the north (the north face of the shape), not the view of the shape while facing north.

Summer 1 – Sequences Recap — *MATHSLINK 8C CHAPTER 10* — *4 lessons*

Key concepts (GCSE subject content statements) — **The Big Picture**: Algebra progression map

- generate terms of a sequence from either a term-to-term or a position-to-term rule
- deduce expressions to calculate the nth term of linear sequences

Return to overview

Possible themes	Possible key learning points
• Explore sequences	• Generate terms of a sequence from a position-to-term rule • Find the nth term of an ascending linear sequence • Find the nth term of an descending linear sequence • Use the nth term of a sequence to deduce if a given number is in a sequence

Prerequisites	Mathematical language	Pedagogical notes
• Use a term-to-term rule to generate a sequence • Find the term-to-term rule for a sequence • Describe a sequence using the term-to-term rule	Sequence Linear Term Difference Term-to-term rule Position-to-term rule Ascending Descending **Notation** T(n) is often used when finding the nth term of sequence	Using the nth term for times tables is a powerful way of finding the nth term for any linear sequence. For example, if the pupils understand the 3 times table can ne described as '3n' then the linear sequence 4, 7, 10, 13, … can be described as the 3 times table 'shifted up' one place, hence 3n + 1. Exploring statements such as 'is 171 is in the sequence 3, 9, 15, 21, 27, ..?' is a very powerful way for pupils to realise that 'term-to-term' rules can be inefficient and therefore 'position-to-term' rules (nth term) are needed. NCETM: Algebra NCETM: Glossary **Common approaches** *Teachers refer to a sequence such as 2, 5, 8, 11, … as 'the three times table minus one', to help pupils construct their understanding of the nth term of a sequence.* *All students have the opportunity to use spreadsheets to generate sequences*
Reasoning opportunities and probing questions	**Suggested activities**	**Possible misconceptions**

• Show me a sequence that could be generated using the nth term 4n ± c. And another. And another … • What's the same, what's different: 4, 7, 10, 13, 16, …. , 2, 5, 8, 11, 14, … , 4, 9, 14, 19, 24, …. and 4, 10, 16, 22, 28, …? • The 4th term of a linear sequence is 15. Show me the nth term of a sequence with this property. And another. And another … • Convince me that the nth term of the sequence 2, 5, 8, 11, … is 3n -1 . • Kenny says the 171 is in the sequence 3, 9, 15, 21, 27, … Do you agree with Kenny? Explain your reasoning.	KM: Spreadsheet sequences KM: Generating sequences KM: Brackets and sequences KM: Maths to Infinity: Sequences KM: Stick on the Maths: Linear sequences NRICH: Charlie's delightful machine NRICH: A little light thinking NRICH: Go forth and generalise **Learning review** KM: 8M9 BAM Task	• Some pupils will think that the nth term of the sequence 2, 5, 8, 11, … is n + 3. • Some pupils may think that the (2n)th term is double the nth term of a linear sequence. • Some pupils may think that sequences with nth term of the form 'ax ± b' must start with 'a'.

Summer 2 – Algebra | *MATHSLINK 8C CHAPTER 5 & 13* | *5 lessons*

Key concepts (GCSE subject content statements)

The Big Picture: Algebra progression map

- understand and use the concepts and vocabulary of identities
- know the difference between an equation and an identity
- simplify and manipulate algebraic expressions by expanding products of two binomials and factorising quadratic expressions of the form $x^2 + bx + c$
- argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments
- translate simple situations or procedures into algebraic expressions or formulae

Return to overview

Possible themes	Possible key learning points
• Understand equations and identities • Manipulate algebraic expressions • Construct algebraic statements	• Understand the meaning of an identity • Multiply two linear expressions of the form (x + a)(x + b) • Multiply two linear expressions of the form (ax + b)(cx + d) • Expand the expression $(x + a)^2$ • Factorise a quadratic expression of the form $x^2 + bx$ • Factorise a quadratic expression of the form $x^2 + bx + c$ • Work out why two algebraic expressions are equivalent • Create a mathematical argument to show that two algebraic expressions are equivalent • Distinguish between situations that can be modelled by an expression or a formula • Create an expression or a formula to describe a situation

Prerequisites	Mathematical language	Pedagogical notes

• Manipulate expressions by collecting like terms • Know that $x \times x = x^2$ • Calculate with negative numbers • Know the grid method for multiplying two two-digit numbers • Know the difference between an expression, an equation and a formula	Inequality Identity Equivalent Equation Formula, Formulae Expression Expand Linear Quadratic **Notation** The equals symbol '=' and the equivalency symbol '≡'	In the above KLPs for factorising and expanding, a, b, c and d are positive or negative. Students should be taught to use the equivalency symbol '≡' when working with identities. During this unit students could construct (and solve) equations in addition to expressions and formulae. See former coursework task, opposite corners NCETM: Algebra NCETM: Departmental workshops: Deriving and Rearranging Formulae NCETM: Glossary **Common approaches** *All students are taught to use the grid method to multiply two linear expressions. They then use the same approach in reverse to factorise a quadratic.*
Reasoning opportunities and probing questions	**Suggested activities**	**Possible misconceptions**
• The answer is $x^2 + 10x + c$. Show me a possible question. And another. And another … (Factorising a quadratic expression of the form $x^2 + bx + c$ can be introduced as a reasoning activity: once students are fluent at multiplying two linear expressions they can be asked 'if this is the answer, what is the question?') • Convince me that $(x + 3)(x + 4)$ does not equal $x^2 + 7$. • What is wrong with this statement? How can you correct it? $(x + 3)(x + 4) \equiv x^2 + 12x + 7$. • Jenny thinks that $(x - 2)^2 = x^2 - 4$. Do you agree with Jenny? Explain your answer.	KM: Stick on the Maths: Multiplying linear expressions KM: Maths to Infinity: Brackets KM: Maths to Infinity: Quadratics NRICH: Pair Products NRICH: Multiplication Square NRICH: Why 24? **Learning review** KM: 9M2 BAM Task, 9M3 BAM Task	• Once students know how to factorise a quadratic expression of the form $x^2 + bx + c$ they might overcomplicate the simpler case of factorising an expression such as $x^2 + 2x$ ($\equiv (x + 0)(x + 2)$) • Many students may think that $(x + a)^2 \equiv x^2 + a^2$ • Some students may think that, for example, $-2 \times -3 = -6$ • Some students may think $x^2 + 12 + 7x$ is not equivalent to $x^2 + 7x + 12$, and therefore think that they are wrong if the answer is given as $x^2 + 7x + 12$

Summer 2 – Algebra , Solving equations and inequalities | ***MATHSLINK 9C*** | *3 lessons*

Key concepts (GCSE subject content statements)

The Big Picture: Algebra progression map

- understand and use the concepts and vocabulary of inequalities
- solve linear inequalities in one variable
- represent the solution set to an inequality on a number line

Return to overview

Possible themes	Possible key learning points
• Explore the meaning of an inequality • Solve linear inequalities	• Find the set of integers that are solutions to an inequality, including the use of set notation • Know how to show a range of values that solve an inequality on a number line • Solve a simple linear inequality in one variable with unknowns on one side • Solve a complex linear inequality in one variable with unknowns on one side • Solve a linear inequality in one variable with unknowns on both sides • Solve a linear inequality in one variable involving brackets • Solve a linear inequality in one variable involving negative terms • Solve problems by constructing and solving linear inequalities in one variable

Prerequisites	Mathematical language	Pedagogical notes

• Understand the meaning of the four inequality symbols • Solve linear equations including those with unknowns on both sides	(Linear) inequality Unknown Manipulate Solve Solution set Integer **Notation** The inequality symbols: < (less than), > (greater than), ≤ (less than or equal to), ≥ (more than or equal to) The number line to represent solutions to inequalities. An open circle represents a boundary that is not included. A filled circle represents a boundary that is included. Set notation; e.g. {-2, -1, 0, 1, 2, 3, 4}	The mathematical process of solving a linear inequality is identical to that of solving linear equations. The only exception is knowing how to deal with situations when multiplication or division by a negative number is a possibility. Therefore, take time to ensure students understand the concept and vocabulary of inequalities. NCETM: Departmental workshops: Inequalities NCETM: Glossary **Common approaches** *Students are taught to manipulate algebraically rather than be taught 'tricks'. For example, in the case of -2x > 8, students should not be taught to flip the inequality when dividing by -2. They should be taught to add 2x to both sides. Many students will later generalise themselves. Care should be taken with examples such as 5 < 1 – 4x < 21 (see reasoning opportunities).*
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
• Show me an inequality (with unknowns on both sides) with the solution x ≥ 5. And another. And another … • Convince me that there are only 5 common integer solutions to the inequalities 4x < 28 and 2x + 3 ≥ 7. • What is wrong with this statement? How can you correct it? 1 – 5x ≥ 8x – 15 so 1 ≥ 3x – 15. • How can we solve 5 < 1 – 4x < 21 ? For example, subtracting 1 from all three parts, and then adding 4x, results in 4 + 4x < 0 < 20 + 4x. This could be broken down into two inequalities to discover that x < -1 and x > -5, so -5 < x < -1. The 'trick' (see common approaches) results in the more unconventional solution -1 > x > -5.	KM: Stick on the Maths: Inequalities KM: Convinced?: Inequalities in one variable NRICH: Inequalities	• Some students may think that it is possible to multiply or divide both sides of an inequality by a negative number with no impact on the inequality (e.g. if -2x > 12 then x > -6) • Some students may think that a negative x term can be eliminated by subtracting that term (e.g. if 2 – 3x ≥ 5x + 7, then 2 ≥ 2x + 7) • Some students may know that a useful strategy is to multiply out any brackets, but apply incorrect thinking to this process (e.g. if 2(3x – 3) < 4x + 5, then 6x – 3 < 4x + 5)

Summer 2 – Algebra , Solving equations and inequalities	MATHSLINK 9C	4 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Algebra progression map

- solve, in simple cases, two linear simultaneous equations in two variables algebraically
- derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution
- find approximate solutions to simultaneous equations using a graph

Return to overview

Possible themes	Possible key learning points
• Solve simultaneous equations • Use graphs to solve equations • Solve problems involving simultaneous equations	• Understand that there are an infinite number of solutions to the equation ax + by = c (a ≠ 0, b ≠ 0) • Find approximate solutions to simultaneous equations using a graph • Solve two linear simultaneous equations in two variables in very simple cases (addition but no multiplication required) • Solve two linear simultaneous equations in two variables in very simple cases (subtraction but no multiplication required) • Solve two linear simultaneous equations in two variables in very simple cases (addition or subtraction but no multiplication required) • Solve two linear simultaneous equations in two variables in simple cases (multiplication of one equation only required with addition) • Solve two linear simultaneous equations in two variables in simple cases (multiplication of one equation only required with subtraction) • Solve two linear simultaneous equations in two variables in simple cases (multiplication of one equation only required with addition or subtraction) • Derive and solve two simultaneous equations • Solve problems involving two simultaneous equations and interpret the solution

Prerequisites	Mathematical language	Pedagogical notes
• Solve linear equations • Substitute numbers into formulae • Plot graphs of functions of the form y = mx + c, x ± y = c and ax ± by = c) • Manipulate expressions by multiplying by a single term	Equation Simultaneous equation Variable Manipulate Eliminate Solve Derive Interpret	Students will be expected to solve simultaneous equations in more complex cases in Stage 10. This includes involving multiplications of both equations to enable elimination, cases where rearrangement is required first, and the method of substitution. NCETM: Glossary **Common approaches** *Students are taught to label the equations (1) and (2), and label the subsequent equation (3)* *Teachers use graphs (i.e. dynamic software) to demonstrate solutions to simultaneous equations at every opportunity*

Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
• Show me a solution to the equation 5a + b = 32. And another, and another … • Show me a pair of simultaneous equations with the solution x = 2 and y = -5. And another, and another … • Kenny and Jenny are solving the simultaneous equations x + 4y = 7 and x – 2y = 1. Kenny thinks the equations should be added. Jenny thinks they should be subtracted. Who do you agree with? Explain why.	KM: Stick on the Maths ALG2: Simultaneous linear equations NRICH: What's it worth? NRICH: Warmsnug Double Glazing NRICH: Arithmagons **Learning review** KM: 9M5 BAM Task	• Some students may think that addition of equations is required when both equations involve a subtraction • Some students may not multiply all coefficients, or the constant, when multiplying an equation • Some students may think that it is always right to eliminate the first variable • Some students may struggle to deal with negative numbers correctly when adding or subtracting the equations

Summer 2 – Assessment	***3 lessons***
• One/Two hour non calculator and calculator SAT style tests • Self-assessment sheets completed • Review and self-assessment of performance stuck into books	